How Do I Know if a Number is Prime?

By Victoria Amarasiri, 5/2017

Why Is It Important To Know Prime Numbers?

When you can recognize that a number is prime, you save time by not trying to find things to divide into it.

Understanding primes is foundational to understanding numbers.

You will solve middle-school math problems with greater speed and accuracy when you quickly recognize a number as prime.

Prime numbers come up lots in 6-8th grade math.

How Do I Know if a Number is Prime?

A good way to train yourself to identify a prime number is to **memorize the smaller prime numbers**.

We cannot possibly memorize all primes - there are an infinite number of prime numbers...

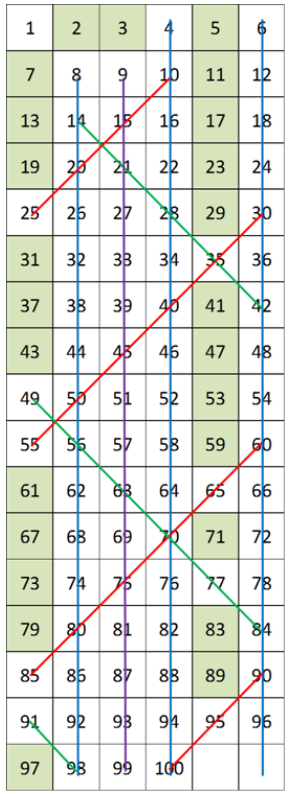
But we can memorize all primes under 100; there are only 25!

As of May 2017, the largest known prime number is $2^{74,207,281} - 1$, a number with 22,338,618 digits. And that is a small number when you are looking at infinity!

How Do I Know if a Number is Not Prime? –Eratosthenes Sieve

Over 2000 years ago, Eratosthenes (271–194 B.C) an Alexandrian geographer and astronomer and friend of Archimedes devised a way to list prime numbers with the invention of his famous sieve.

More recently, another “sieve” has been devised to “sift” out the primes in the first 100 natural numbers that are arranged into the 6 x 17 rectangular array as shown.



- Cross out multiples of 2 and 3 with vertical lines, (except the numbers 2 and 3).
- Cross out multiples of 5, (except 5) with diagonal lines sloping down.
- Cross out multiples of 7, (except 7) with diagonal lines sloping up.
- The remaining green colored squares are the primes.

How to Memorize Primes – A Prime Story

A wonderful approach to retaining prime numbers is to learn them through a story that is easy to recite. Below is one example, “The Math Club”. You can create your own!

At **2**:**35**, I went to the **7**-**11** on Friday the **13**th.

The cashier thought I looked **17**, I am **19**, and would not let me purchase a lottery ticket.

So I purchased **23** candy bars for my sister who is **29** and holds a math club with **31** students and needed some prizes for a math contest.

Children that solved **37** out of **41** problems correctly would get a candy bar.

Everyone finished the contest in **43** minutes; **47**% won candy, but **53**% did not.

My sister gave the remaining candy to my grandpa, who is **59** years old and loves chocolate, but not to my **61** year old grandma who had just lost **67** pounds.

Children that scored between **71** - **73** % were asked to re-memorize their prime numbers.

Her goal is for **79**% of the club to learn their primes up to **83** or **89** so that the club will do better on the next math contest.

And congratulations to you for getting a **97**%!

How to Memorize Primes – A Prime Picture

Use a visual approach to memorizing primes -- a picture.

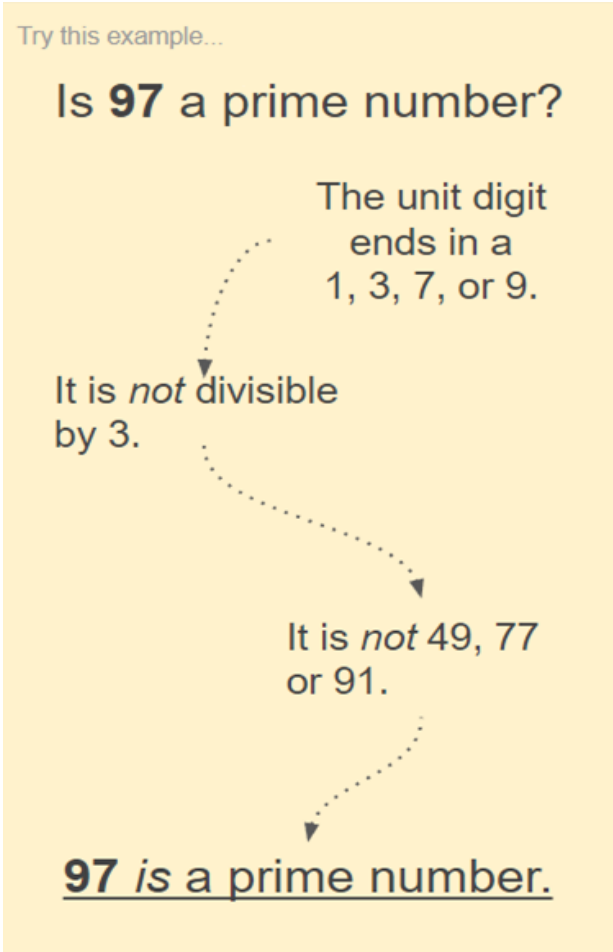
You can draw one too!

A Shortcut to Find Primes from 10 to 100

This shortcut is wonderful for identifying prime numbers from 10 to 100.

A number is prime if it meets these 3 conditions:

- Its unit digit is a 1, 3, 7 or 9.
- It is *not* divisible by 3.
  (i.e. The sum of its digits is not divisible by 3.)
- It is not 49, 77 or 91.



How to Memorize Primes – By 10s Place Value

Questions on prime numbers may ask “How many primes are there between x and y?” It is helpful to remember how many primes are in each tens group under 100.

How many prime numbers are there under 20?

How many prime numbers are between 50 and 60?

Is 23 a prime number?

What is the largest prime number under 100?

How many primes?	Primes				
	Tens	Units			
4	0	2	3	5	7
4	1	11	13	17	19
2	2		23		29
2	3	31		37	
3	4	41	43	47	
2	5		53		59
2	6	61		67	
3	7	71	73		79
2	8		83		89
1	9			97	

How Will You Memorize Your Primes?

If you decide to take the Memorize Your Primes Challenge, think of how you will memorize the 25 prime numbers under 100:

Use the Eratosthenes sieve to become familiar with primes.

Use the Shortcut, to quickly surface prime numbers under 100.

Memorize. There are only 25!
Start up to 40, then go to 100. Or group and memorize by the tens digit -- handy for questions that ask "How many primes are there between two numbers?"

Draw a picture.

Memorize a Prime Number story.

Play a Prime Numbers App (e.g. Ninja Chicken)

Different approaches work for different people. Use an approach that works best for you!

There is no limit. Prime Numbers up to 1000

	2	3	5	7	11	13	17	19	23
29	31	37	41	43	47	53	59	61	67
71	73	79	83	89	97	101	103	107	109
113	127	131	137	139	149	151	157	163	167
173	179	181	191	193	197	199	211	223	227
229	233	239	241	251	257	263	269	271	277
281	283	293	307	311	313	317	331	337	347
349	353	359	367	373	379	383	389	397	401
409	419	421	431	433	439	443	449	457	461
463	467	479	487	491	499	503	509	521	523
541	547	557	563	569	571	577	587	593	599
601	607	613	617	619	631	641	643	647	653
659	661	673	677	683	691	701	709	719	727
733	739	743	751	757	761	769	773	787	797
809	811	821	823	827	829	839	853	857	859
863	877	881	883	887	907	911	919	929	937
941	947	953	967	971	977	983	991	997	

How You Can Help Your Child?

Math Club is 1 hour a week; of that, no more than 5 minutes is spent on learning a new math area. Prime Number concepts could use additional reinforcement. The club could use your help:

- ➔Spend 10 minutes becoming familiar with primes < 100.
  A couple of days a week. Choose your child's method -- memorization, learning the shortcut, using the Ninja Chicken app or learning the Prime Number Story.

- ➔Spend 10 minutes solving the prime number problems.
  Once your child has a good recall of the prime numbers under 100, try answering some of the questions on the drill sheet printouts provided in club (note: the drill sheets contain questions that for copyright reasons cannot be disseminated electronically).

Please no worries if you cannot!

The SVMS Math Club will always be working on Number Sense.

Start the drumbeat if or when it makes sense for you and your child.

Thank You!

Further information can be found at:

More information is attached after this slide.

SVMS Math Club - An Introduction to Prime Numbers (Please ask for a copy).

Prime Numbers Introduction: http://www.glennwestmore.com.au/introduction/

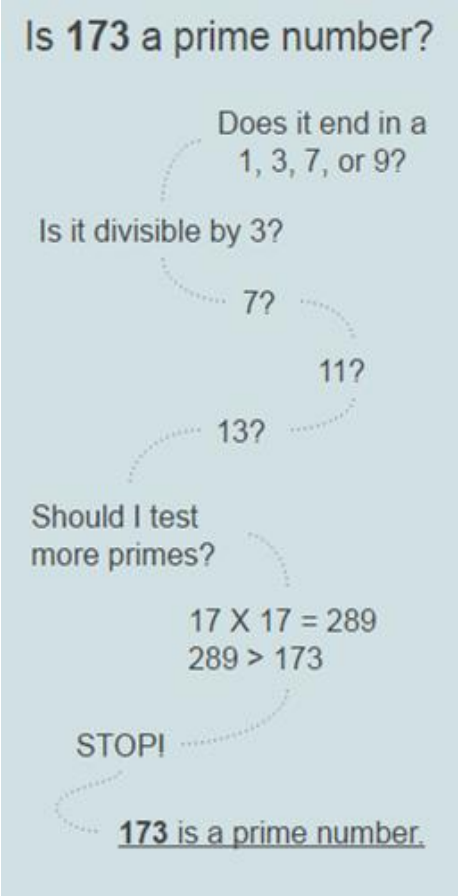
Geek.com - "Why Should We Care About Prime Numbers" https://www.geek.com/news/geek-answers-why-should-we-care-about-prime-numbers-1574269/

Numberphile - Why 1 is Not a Prime: https://www.youtube.com/watch?v=_zWxVSQOxyk

And more for the very curious....

A Good Way To Know if Any Number is Prime

Determine if a number is prime by checking if the number has a prime factor other than itself and 1.



Start from the smallest prime factors (i.e. 2, 3, 5, …) and work our way up, checking if the prime is a factor of the number. If a factor is found, the number is a composite.

Continue until the largest prime is reached that could possibly be a factor of the number (i.e. The largest possible prime factor is greater than the number when the prime factor is multiplied by itself or continue until the resulting quotient is less than the divisor.)

Use divisibility rules to quickly check each prime as a possible factor of the number.

Discard numbers when the unit digit is not 1, 3, 7 or 9.

What are the Divisibility Rules?

Divisibility rules enable us to check if a prime is a factor of the number with greater speed and accuracy.

Divisor	*Divisibility Rule*
2	A number is divisible by 2 if the unit digit is even.
3	A number is divisible by 3 if the sum of the digits is divisible by 3.
5	A number is divisible by 5 if the unit digit ends in a 0 or a 5.
7	To find out if a number is divisible by seven, take the last digit, double it, and subtract it from the rest of the number. If you get an answer divisible by 7 (including zero), then the original number is divisible by seven. If you don't know the new number's divisibility, you can apply the rule again.
11	For three digit numbers, a number is divisible by eleven if the outer digits sum to the middle digit. For three digit numbers and larger,take every other digit of the number and add them up. Now take the digits you didn't use and add them up. Now subtract these two sums from each other. If the difference between the two sums is divisible by 11, then the number is divisible by 11.
13	Subtract 9 times the units digit from the rest of the number. The number is divisible by 13 if the result is divisible by 13.
17	Subtract 5 times the last digit from the rest of the number. The number is divisible by 17 if the result is divisible by 17.
19	Add two times the last digit to the remaining leading truncated number. The number is divisible by 19 if the result is divisible by 19.

The Shortcut – How Does it Work?

A prime number above 10 is never even, so we eliminate any number that ends in an even digit. A prime is never a multiple of 5, so we eliminate any number that ends in a 5. Therefor, the only possible unit digits that a prime above 10 can have are 1, 3, 5 and 7.

Next we sift through the prime factors to prove that the number is not a composite. 2 was already removed as a possible factor. We next check if 3 is a prime factor using its divisibility rule (i.e. a number is a multiple of 3 if the sum of its digits are divisible by 3.) If the sum of the number's digits are divisible by 3, the number is not a prime.

The number 5 was already removed as a possible factor. We next check if the number is a multiple of 7. There are only three multiples of 7 that haven't already been eliminated. They are 49, 77, and 91. I will leave that for you to prove. These are easy to spot and remember. If the number is a 49, 77 or 91, it is a multiple of 7 and can be discarded.

We are done. The next prime factor is 11 and we know we can stop here because we have checked all lesser prime factors that could be paired with 11 and 11 X 11 is greater than 100.

Short Cut

Is **97** a prime number?

Is **97** a prime number?

The unit digit ends in a 1, 3, 7, or 9?

It is *not* divisible by 3?

Easy Shortcut - Is the sum of the number's digits is not divisible by 3?

9+7=16 and 16 is not divisible by 3.

It is not 49, 77 or 91.

97 is a prime number.