

So You Think You Can Multiply?

So you think you can multiply? I know you can multiply very well because in elementary school we become very strong at multiplication. But the ways we learn to multiply are often best used with pencil and paper.

When we multiply numbers in our head, we rely on a different set of strategies. Today, we will look at strategies that help us multiply numbers in our heads.

This will require some practice, so please try knowing that nobody is more *mathy* than you! Success is just who practiced the most. Practice 20 times and these strategies may stick for life.

Why multiply in our heads? The thing is, multiplying in our heads makes us better problem solvers. We can retain information while we figure things out and look at all the possible paths. Performing math in our heads, we call this mental math, helps us work faster, avoid hard work and we make less errors.

Here we go!

A List of Multiplication Shortcuts

LEVEL	MULTIPLY WHAT?	USING WHICH SHORTCUTS?
BEGINNER THIRD GRADE MATH CLUB	Multiply by 10	Add a Zero.
	Multiply 2-digit number by 11	Sum Digits and Put in Middle.
	Multiply any 2 numbers	Break Apart.
ADVANCED ELEMENTARY MATH CLUB	Multiply by 4 and 8	Double and Double Again.
	Multiply by 6	Double and Triple.
	Multiply by 5	Add a Zero and Halve.
	Multiply by 12	Add a Zero + Double.
	Multiply by 15	Add a Zero + Halve.
	Multiply by 20	Double and Add a Zero.
	Multiply by 11	Sum Digit Pairs.
MASTER COMPETITION MATH CLUB	Multiply any 2 numbers	Double One, Halve the Other.
	Multiply any 2 numbers	LO+IF.
	Multiply any 2 numbers between 10 and 20	N+U LL.
	Multiply 25, 50, 75, 125	Like Quarters of a Dollar.
	Multiply 2 numbers close to 100 – 99 and 101	99 Number, 100–Number. 101 Number, Number (again).
	Multiply by 111	Sum Digit Triples.



How We Multiply Tens

Can you multiply these numbers in your head? Lets try...

$$10 \times 60 =$$

I think most of us know to simply add a zero.

Multiply the non-zeros. Count up the zeros and add the zeros to the end.

It works like this...

$$10 \times 60 = \text{Multiply the non-zeros.}$$

6

$$6 \times 1 = 6$$

600

Count up and tack on the zeros to the answer.

600

Walaah!

Now try with a friend to multiply using this method:

$$15 \times 10 \text{ Did you get 150?}$$

$$842 \times 10$$

$$41 \times 50$$

$$32 \times 10$$

$$90 \times 100$$

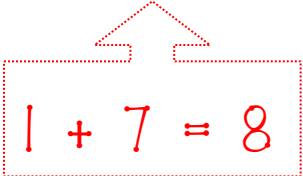
$$6 \times 10^5 \text{ The little 5 means add 5 zeros.}$$


 EXPERIENCE 2

How We Multiply A 2-Digit Number by 11

To multiply a 2-digit number by 11, we add the digits of the number and put the sum in the middle.

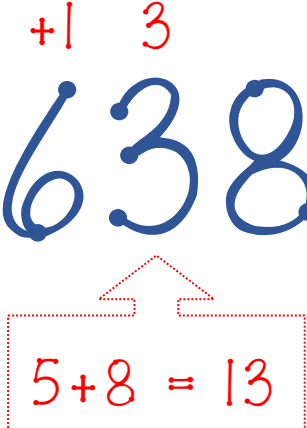
It goes something like this...

$$17 \times 11 = 187$$


$1 + 7 = 8$

Let's try a harder example. Notice that when we add the digits of a larger number and put the sum in the middle, that there is a "carry" of 1.

Here is what we do when that happens...

$$58 \times 11 = 638$$


$5 + 8 = 13$


 TAKEAWAY

How We Multiply a 2-Digit Number by 11

Can you multiply these numbers in your head? Maybe practice this shortcut over two or three days if you can.

Day 1

What is 14×11 ?

(It turns out to be 154. Can you get that answer?)

What is 87×11 ? (Whoa! Remember to add the carry to the tens digit. The answer is 957. Can you get that answer?)

What are these:

$16 \times 11 =$

$35 \times 11 =$

Day 2

$13 \times 11 =$

$11 \times 18 =$

$12 \times 11 =$

$63 \times 11 =$

$76 \times 11 =$

$11 \times 46 =$

Day 3

$11 \times 19 =$

$15 \times 11 =$

$17 \times 11 =$

$53 \times 11 =$

$11 \times 81 =$

$11 \times 99 =$

Challenge: Have a car contest - have your parent or friend call out a number and see who can multiply it by 11 (without pencil or paper!)

EXPERIENCE 3

How We Multiply (and avoid hard work!)

Can you multiply these numbers in your head?

$$6 \times 14 =$$

We know how to multiply... But when we want to multiply in our heads, without pencil and paper, there is usually an easier way to multiply a set of numbers if we look. And we are all about avoiding hard work!

We can often find an easier way to multiply a set of numbers in our heads if we look. There is often a shortcut.

It works like this...

From third grade math, we know that an equation can be represented in many ways. For example:

$$6 \times 14 = 14 + 14 + 14 + 14 + 14 + 14$$

What other ways can think of to represent 6×14 ? Which might you use to solve the problem faster and with less error?

$$14 \times 6$$

Triple 14 and double the result

$$28 + 28 + 28$$

$$14 \times 6$$

$$3 \times 28$$

Ten sixes + four sixes

$$3 \times 2 \times 14$$

$$2 \times 3 \times 2 \times 7$$

$$10 \times 6 + 4 \times 6$$

$$21 \times 2 \times 2$$

EXPERIENCE 4

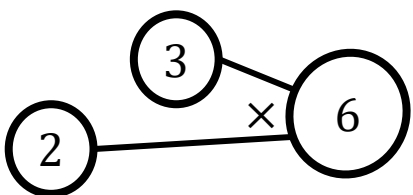
How We Multiply Using BREAK APART

Can you multiply these numbers in your head? Lets try...

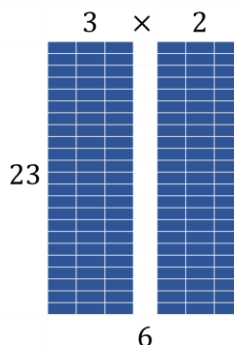
$$6 \times 23 =$$

To solve this, we might be able to multiply in our heads. But there may be an easier way that may be less prone to error.

We can easily multiply any number by 6. Multiplying any number by 6 is the same as multiplying the number by 3×2 .



6 can be broken down into 2×3 .



We can multiply 23 by 6 or we can multiply 23 by 3 and then by 2.

It works like this...

$$6 \times 23 = 23 \times 3 \times 2$$

Triple Double

$$23 \quad 46 \quad 69 \quad 138 \quad \boxed{138}$$

Multiply by 6 Shortcut: Triple the number and double the result!

TAKEAWAY

How We Multiply

Using **PART PART WHOLE**

Try with a friend to multiply using the easier method:

$$6 \times 34 \quad 34 \ 68 \ 102 \ 204$$

$$6 \times 18$$

$$4 \times 32 \quad \text{What parts make up 4?}$$

$$24 \times 6$$

$$35 \times 6$$

$$4 \times 16$$

$$16 \times 8 \quad \text{What parts make up 8?}$$

Never use pencil or paper! Solve in your head.

EXPERIENCE 4

How We Multiply Using **BREAK APART**

Can you multiply these numbers in your head? Lets try...

$$12 \times 24 =$$

To solve this, most of us will write out a stacked multiplication problem and solve using long multiplication. But there is a shortcut that we picked up in school that can help us do this in our heads.

We can easily multiply numbers in our head using **Break Apart**. We break apart one number into two friendlier numbers (e.g. tens digit and unit digit).

It works like this...

Break Apart 12 into 10 and 2

$$12 \times 24 = 10 \times 24 + 2 \times 24$$

$$240 + 48$$

All of this
can be done
in our heads!

288


TAKEAWAY


How We Multiply Using **BREAK APART**

Can you multiply these numbers in your head? Try this with a friend. Maybe practice this shortcut at home if you can.

$$15 \times 60 \quad 10 \times 60 + 5 \times 60 \dots \text{or } 10 \times 60 + \text{half the result!}$$

$$23 \times 12$$

$$20 \times 8 \quad \text{How should we break apart 20?}$$

$$22 \times 12$$

$$15 \times 16$$

$$97 \times 101 \quad \text{How should we break apart 101?}$$

900, 276, 160, 264, 240, 9797

Never use pencil or paper! Solve in your head.

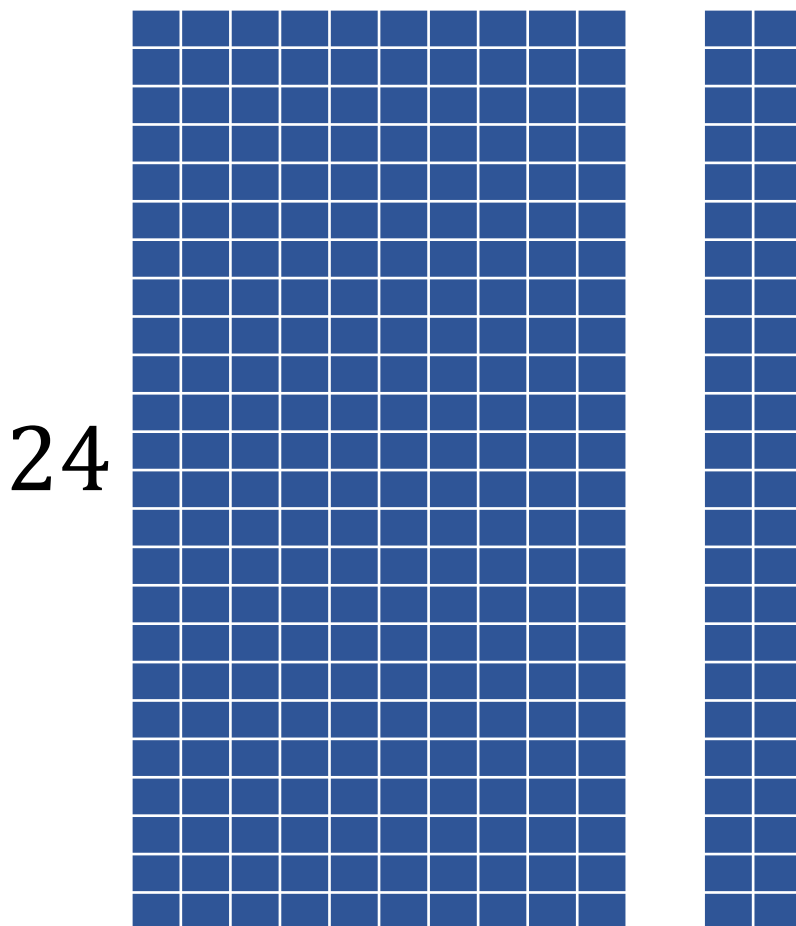
EXPERIENCE 4

How We Multiply Using **BREAK APART**

Can you multiply these numbers in your head? Lets try...

$$12 \times 24 =$$

$$10 \quad + \quad 2$$



EXPERIENCE 4

How We Multiply Using Part Part Whole

We can also break apart using the factors of a number.

$$35 \times 6 =$$

To solve this, most of us can do this in our heads but we may have noticed from experience that we do so with some error. The problem $35 \times 2 \times 3$ is an easier one to solve.

Break Apart the smaller number into its factors and multiply.

It works like this...

$$35 \times 6 = 6 = 2 \times 3.$$

Double 35 First multiply by 2
(i.e. double 35.)

70 × 3 Multiply the result by 3.

210

Walaah!

Hint:
Always
look
for the
easiest
ordering.

Now try with a friend to multiply using either break apart method:

15×60 Did you double and triple and add a 0 to the end?

23×12 Break apart units and tens. 32×8 Double 32 3 times!

97×101

22×12

121×121

EXPERIENCE 5

How We Multiply a Number by 4, 6 and 8

Can you multiply this number pair in your head?

$$37 \times 4 =$$

Multiplying a number by 4, 6 or 8 is pretty easy. Yet you may have noticed from club that we do get these wrong from time to time. Here is a speedier approach that tends to avoid the mistakes we make on these easier problem types.

The shortcut to multiply by 4 is: **Double and Double Again.**

It works like this...

$$37 \times 4 = 37 \times 2 \times 2$$

Is the same as

$$37 \times 4 = 74 \times 2 = 148$$

Double! Double again!

The shortcut to multiply by 6 is: **Double and Triple.**

It works like this...

$$35 \times 6 = 35 \times 2 \times 3$$

Is the same as

$$35 \times 6 = 70 \times 3 = 210$$

Double! Triple!

Numbers are easier to work with if we break them apart - into factors, into tens & units, into friendly numbers. Don't get stuck on the problem as it is written - avoid the hard work and look for the easier way!

TAKEAWAY

How We Multiply a Number by 4, 6 and 8

Try with a friend to multiply using the easier method:

$$4 \times 36 \quad 72 \dots 144$$

$$32 \times 8 \quad \text{Double three times!}$$

$$54 \times 4$$

$$45 \times 6$$

$$65 \times 6$$

$$45 \times 8$$

$$16 \times 8$$

$$55 \times 6$$

$$32 \times 4$$



How We Multiply a Number by 5

Can you multiply a number by 5 in your head? Lets try...

$$38 \times 5 =$$

Its not too hard, most of us can multiply this one in our head. But when we want speed, we don't see this as a 5. We see the 5 as a 10 halved. When we look at the problem this way – we realize we can simply add a 0 to the number and halve it to get the result.

The shortcut to multiply by 5 is: **Add a Zero and Halve.**

The way this works...

$$38 \times 5 = 380 \quad \begin{array}{l} \text{Add a zero} \\ \downarrow \end{array} \quad \begin{array}{l} \text{Halve} \\ \downarrow \end{array} \quad \boxed{190}$$

$$\frac{10}{2}$$

Now try with a friend using the speedier method:

26×5 It goes like this... 260 ... 130.

43×5 430 ... 215.

28×5

31×5

84×5

67×5

49×5

44×5

$80 \div 5$ What would the shortcut for this problem be?

$135 \div 5$

The answers are 130, 215, 140, 155, 420, 335, 245, 220, 16, 27.

EXPERIENCE 7

How We Multiply by 12 Using Break Apart

Can you multiply these numbers in your head?

$$12 \times 24 =$$

To solve this, most of us will write out a stacked multiplication problem and solve using long multiplication. But Break Apart makes this much easier to solve. There are some additional shortcuts to take.

Add a Zero and Double. To multiply by 12, we extend on Break Apart. 1) Take the number and add a zero to the end. 2) Then take the number and double it. 3) Add the two results to get the answer.

This is the way it works...

$$12 \times 24 = 240 + 48 = 288$$

Add a zero Double
↓ ↓

Hint: If doubling is too hard, you could keep a running total. E.g. 240, 264, 288

Now try with a friend to multiply using this break apart method:

$$12 \times 60 \quad 600 + 120 = 720$$

$$45 \times 12$$

$$12 \times 53$$

$$18 \times 12$$

$$81 \times 12 \quad 810, 891, 972$$

$$99 \times 12$$

The answers are 720, 540, 216, 636, 216, 972, 1188.

EXPERIENCE 8

How We Multiply by 15 Using Break Apart

Can you multiply these numbers in your head?

$$15 \times 24 =$$

To solve this, most of us will write out a stacked multiplication problem and solve using long multiplication. But Break Apart makes this much easier to solve. There are some additional shortcuts to take.

Add a Zero and Halve. To multiply by 15, we extend on Break Apart. 1) Take the number and add a zero to the end. 2) Take that result and halve it. 3) Add the two results to get the answer.

This is the way it works...

$$15 \times 24 = 240 + 120 = 360$$

Add a zero ↓ Halve ↓
↗

Hint: Keeping a running total may be easier.

Now try with a friend to multiply using this break apart method:

$$15 \times 60 \quad 600 + 300 = 900$$

$$45 \times 15 \quad 450, 675$$

$$15 \times 53$$

$$18 \times 15$$

$$81 \times 15$$

$$99 \times 15 \quad 990 + 450 + 45 = 1390, 1440, 1485$$

The answers are 900, 675, 795, 270, 1215, 1485

EXPERIENCE 9

How We Multiply by 20 Using Break Apart

Can you multiply these numbers in your head?

$$20 \times 24 =$$

This shortcut comes natural and you may already be using this, but lets make sure.

Double and Add a Zero. To multiply by 20, double the number and add a zero to the end.

This is the way it works...

$$20 \times 24 = 48 \quad \xrightarrow{\text{Add a Zero}} \quad 480$$

Diagram illustrating the process: The number 48 is shown in green. A red dashed arrow labeled "Double" points down to 48. A red dashed arrow labeled "Add a Zero" points from 48 to the boxed number 480, which is also in green.

Now try with a friend to multiply using this break apart method:

20×60 120, 1200

45×20

20×53

20×15

81×20

99×15 200-2=198, 1980

The answers are 1200, 900, 1060, 300, 1620, 1980



How We Multiply Any Number by 11

To multiply any number by 11, we Sum Digit Pairs starting from right to left.

It kind of works like a 2 car train catching 2 digits at a time and summing them up along the way!

$$312 \times 11 =$$

Units digit	312	_ _ _ 2
	_ _ _	
Tens + units digits	312	_ _ _ 32
	_ _ _	
Hundreds + tens digits	312	_ _ _ 432
	_ _ _	
Hundreds digit	312	_ _ _ 3432
	_ _ _	

You might want to try this out with a friend:

11×62	11×38 There is a carry in this problem!
602×11	11×254

Answers: 682, 418, 6622, 2794.

EXPERIENCE II

Double One, Half the Other

Can you multiply these numbers in your head? Lets try...

$$35 \times 18 =$$

Would you agree that multiplying 70×9 is a much easier problem? Did you know that they both problems have the same answer? It is sometimes easier to double one number and halve the other.

Double one number and halve the other to get to an easier problem to solve.

It works like this...

$$35 \times 18 =$$

Double 35 and halve 18.

$$(35 \times 2) \times (18 \times \frac{1}{2}) =$$

Note the $2 \times \frac{1}{2} = 1$
They cancel each other out!
Leaving you with an easy multiplication problem to solve.

$$70 \times 9 =$$

630

Hint: You are looking for a way to get to a friendly pair of numbers to multiply.

Now try with a friend to multiply using this speedy method:

$$15 \times 28$$

Did you double the 15 and halve the 28 – you are on the right track!

$$32 \times 45$$

$$16 \times 26$$

$$75 \times 52$$

$$32 \times 42$$

EXPERIENCE 12

LOIF - Last Outer + Inner First

How We Multiply a Pair of 2 Digit Numbers

Can you multiply a pair of two-digit numbers in your head?

$$34 \times 26 =$$

When there are no faster shortcuts, LOIF is a good strategy.

It works like this...

$$34 \times 26$$

Last
We multiply the last digits.
Write down the units digit.

$$4 \times 6 = 24$$

If there is a carry, store it on the fingers of your left hand (or right if you are a lefty).

$$34 \times 26$$

Outer + Inner
Next we multiply the outer and inner digits and add the results together. Add in the digit you stored on the fingers of your left hand.

$$3 \times 6 + 4 \times 2 + 2 = 28$$

Write down the tens digit. If there is a carry, store it on your fingers again.

$$34 \times 26$$

First
Multiply the first digits. Add in the carry in write down this number to the left of the tens digit.

$$3 \times 2 + 2 = 8$$

884


 TAKEAWAY

LOIF - Last Outer + Inner First

How We Multiply a Pair of 2 Digit Numbers

Now try with a friend to multiply using LOIF:

Set 1

$82 \times 23 =$ Did you get to the answer 1886?

$26 \times 73 =$

$89 \times 27 =$

$85 \times 59 =$

Set 2

$47 \times 74 =$

$41 \times 85 =$

$41 \times 62 =$

93×37

Set 3

$54 \times 89 =$

$84 \times 36 =$

$44 \times 18 =$

$53 \times 58 =$

Set 3: 4806, 3024, 792, 3074
 Set 2: 3478, 3485, 2542, 3441
 Set 1: 1886, 1898, 2403, 5015

Answer Key:



How We Multiply by 25

Can you multiply these numbers in your head? Lets try...

$$21 \times 25 =$$

How many quarters are in a dollar? Four. There are also four 25s in every 100.

Like Quarters of a Dollar, divide the number by 4. The result is the first digit or digits. If the remainder is 1, the last 2 digits are 25. For 2 its 50. For 3 its 75. For 0 its 00.

It works like this...

$21 \times 25 = 21$
 $21 \div 4 = 5R1$
525

R0	00
R1	25
R2	50
R3	75

Now try with a friend to multiply using the shortcut:

24×25 6 with no remainder... 600

25×28

25×49

87×25

32×25



How We Multiply by 50

Can you multiply these numbers in your head? Lets try...

$$21 \times 50 =$$

How many half-dollars are in a dollar? Two. There are also two 50s in every 100.

Like Half-Dollars of a Dollar, divide the number by 2. The result is the first digit or digits. If the remainder is 1, the last 2 digits are 50. For 0 its 00.

It works like this...

$21 \times 50 = 21$
 \Rightarrow
 1050

$21 \div 2 = 10R1$

R0	00
R1	50

Now try with a friend to multiply using the shortcut:

24×50 12 with no remainder... 1200

50×28

50×49

87×50

32×50






EXPERIENCE IS

How We Multiply Any Number by 111

To multiply any number by 11, we Sum Digit Triples starting from right to left.

It kind of works like a 3 car train catching 3 digits at a time and summing them up along the way!

$$312 \times 111 =$$

Units digit	312	_____2
		
Tens + units digits	312	_____32
		
Hundreds + tens + units digits	312	_____632
		
Hundreds + tens digits	312	_____4632
		
Hundreds digit	312	_____34632
		

You might want to try this out with a friend:

111×18

29×111 There is a carry in this problem!

254×111

111×602