

SIX STRATEGIES FOR ADDING IN OUR HEADS

So you think you can add? I know you can add and very well because in elementary we become very strong at adding big numbers. But the ways we learn to add are often best used with pencil and paper.

When you want to add numbers in your head, we need a different set of strategies.

Today we will look at 6 strategies that might help us add long lists of big numbers in our heads. They are named: [Keep a Running Total](#), [Pair Evenly Spaced Numbers](#), [Multiply Repeating Addends](#), [Rely on Friendly Numbers](#), [Use an Easier Order](#) and [Add Repeating Digits](#).

This will require some practice, so please try knowing that nobody is more *mathy* than you! It's just who practices the most. Practice 20 times and the strategies will stick for life.

Why add in our heads? The thing is, adding in our heads makes us better problem solvers. We can retain the information while we are figuring things out and looking at all the possible paths. Performing math in our heads, we call this mental math, helps us work faster, avoid hard work and we tend to make less errors.

[The take-away. If you do not like the problem you are given, change the numbers and make the problem easier!](#)

EXPERIENCE I

Keep A Running Total

Can you add a list of numbers in your head?

$$13 + 10 + 10 + 2 + 21$$

It works like this. We say "13, 23, 33, 35, 56." The answer is 56.

It can be easier to total the units and then the tens. This is called the Break-Apart method and can help you keep a running total.

It works like this.

$$1 \begin{array}{|l} \hline 3 \\ \hline \end{array} + 1 \begin{array}{|l} \hline 0 \\ \hline \end{array} + 1 \begin{array}{|l} \hline 0 \\ \hline \end{array} + 2 + 2 \begin{array}{|l} \hline 1 \\ \hline \end{array}$$

Sum up the units digits. 3, 5, 6

1, 2, 3, 5 Sum up the tens digits.

Put the 5 in the tens place and the 6 in the units place.

5 6

Now try with a friend to add these lists, using the easier method:

$$36 + 46 + 22 + 47 + 39$$

Did you get 190 by saying "6, 12, 14, 21, 30" to get a unit of 0 and a carry of 3?

Did you add that carry with the other tens place digits?

And then saying "3, 6, 10, 12, 16, 19" did you get a tens place digit of 19?

$$14 + 33 + 18 + 66$$

$$64 + 35 + 12 + 81$$

$$27 + 34 + 47 + 24$$

$$52 + 13 + 79$$

The answers are 190, 131, 192, 132 and 144.

EXPERIENCE 2

Pair Evenly Spaced Numbers

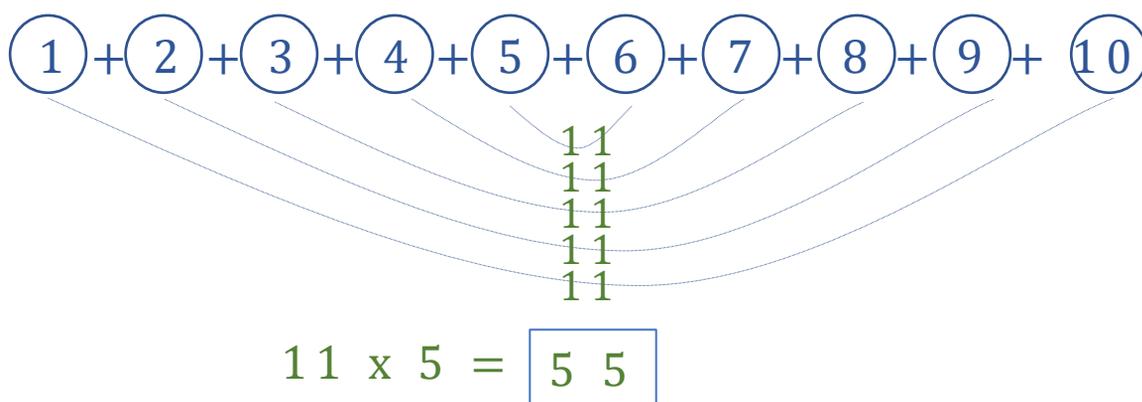
Can you add this list of numbers in your head?

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

Many of us will write this out and add it up or keep a running total.

When the numbers to be added are evenly spaced, we can add the numbers in pairs. If there is a middle number, we can add this in last.

It works like this.



Now try with a friend to add these lists, using the easier method:

$$7 + 11 + 15 + 19 + 23$$

Did you find 2 pairs of 30? Do not forget to add in the remaining middle number.

$$11 + 22 + 33 + 44 + 55 + 66 + 77 + 88$$

It might be easier to use friendly numbers - multiply by 100, then subtract the 15?

$$1 + 3 + 5 + 7 + 9 + 11$$

$$5 + 15 + 25 + 35 + 45$$

The answers are 75, 396, 36 and 125.

EXPERIENCE 3

Multiply Repeating Addends

Can you add this list of numbers in your head?

$$21 + 21 + 21 + 21 + 21 + 21$$

Many of us will write this out and add it up or keep a running total.

The definition of multiplication is repeated addition. It is so much easier to multiply the number by how many times it is repeated.

It works like this.

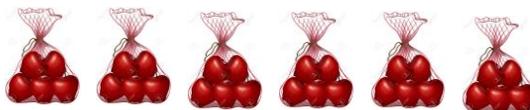
$$21 + 21 + 21 + 21 + 21 + 21$$

$$6 \times 21 = 6 \times 20 + 6 \times 1 = \boxed{126}$$

Now try with a friend to add these lists, using the easier method:

$$5 + 5 + 5 + 5 + 5 + 5$$

Did you count 6 sets of 5s? Did you get 30?



$$11 + 11 + 11 + 11 + 11 + 11 +$$

$$11 + 11 + 11 + 11 + 11 + 11$$

Can't quickly recall 12×11 ? no worries, just practice your math minutes this week.

$$2 + 2 + 2 + 2 + 2$$

$$25 + 25 + 25 + 25 + 25 + 25$$

Think of how much money you have if you have 6 quarters.

The answers are: 30, 132, 10, 150

Rely on Friendly Numbers

Which list of numbers is easier to add in your head?

$$301 + 21 + 198 \quad \underline{\text{OR}}$$

$$300 + 20 + 200 + 1 + 1 - 2$$

$1 + 1 - 2 = 0$... Very easy... And adding the hundreds and tens even easier.

What if I told you the two expressions are the same? Would you agree? Why?

Friendly numbers can make adding easier. We round the number and then we add in extra additions or subtractions to compensate.

It works like this.

$$301 + 21 + 198$$

Round $300 + 20 + 200$ and then compensate $+ 1 + 1 - 2$

520

Note that the compensating numbers happened to equal zero in this problem.

Now try with a friend to add these lists, using this method:

$$106 + 98$$

I might try $100 + 100 + 6 - 2$.

$$93 + 114 + 93$$

$$49 + 49 + 49$$

$$67 + 97 + 21$$

$$99 + 101 + 53$$

$$295 + 397 + 108$$

The answers are: 204, 300, 185, 800, 147, 253

Use an Easier Order

Can you add a list of numbers in your head?

$$85 + 57 + 15$$

Many of us will write this out and add it up or keep a running total, but we can add this in our heads if we look for an easier order.

This problem is simple if we add $85 + 15$ first. Can you see why? A good idea is to look for the easiest pairs to add up first.

It works like this.

$$85 + 57 + 15$$

Look over the problem for pairs that are easy to add up...

$$85 + 15 = 100$$

$$100 + 57$$

... then add in the remaining numbers.

$$157$$

Now try with a friend to add these lists, using the easier method:

$$54 + 36 + 45$$

I might add $54 + 45$ to get 99. Then I can turn that into a friendly number and subtract the compensating 1 from 36. Did you get 135 as your answer? If not, no worries, try again.

$$88 + 28 + 12 \quad \text{Which pair of number is easy to add up? What is the answer?}$$

$$45 + 73 + 27$$

$$37 + 38 + 22 + 40$$

The answers are: 135, 128, 145, 137

Add Repeating Digits

Can you add a list of numbers in your head?

$$234 + 342 + 423$$

Many of us will write this out and add it up or keep a running total.

You may have correctly noticed that each place has the same three numbers. The three numbers are 2, 3, and 4. You can quickly get to the answer after you have summed up the units place.

It works like this.

$$234 + 342 + 423$$

Look over the problem to make sure that the hundreds, tens and units digits are repeating the same numbers.

Add the numbers. Remember that addition is commutative ($1 + 2 + 3 = 3 + 2 + 1$)

$$2 + 3 + 4 = 9$$

Use the result in all places.

999

Now try with a friend to add these lists, using the easier method:

$$11,112 + 11,121 + 11,211 + 12,111 + 21,111$$

The 2 seems to appear one time in each place. Hmmm.

$$11 + 101 + 110$$

$$22 + 33 + 44 + 55$$

It is a little harder when there is a carry involved – but give it a try! Wouldn't adding in pairs also work nicely?

$$1232 + 2321 + 3212 + 2123$$

The answers are: 66666, 222,154, 8888

SIX STRATEGIES FOR ADDING IN OUR HEADS

Keep A Running Total

When the digits are big, some of us find that it can be easier to total the units and then the tens.

How it Works:

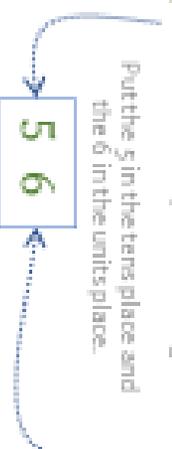
$$13 + 10 + 10 + 2 + 21$$

Sum up the units digits.

$$3, 5, 6$$

1, 2, 3, 5 Sum up the tens digits.

Put the 5 in the tens place and the 6 in the units place.



Rely on Friendly Numbers

Round the number and then add extra additions or subtractions to the problem to compensate.

How it Works:

$$301 + 22 + 198$$

$$300 + 20 + 200$$

and then compensate $+ 1 + 2 - 2$

$$521$$

Pair Evenly Spaced Numbers

When the numbers are evenly spaced, we can add the numbers in pairs. A middle number is added in last.

How it Works:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

$$(1+10) + (2+9) + (3+8) + (4+7) + (5+6) + 5$$



$$11 \times 5 = 55$$

Use an Easier Order

Addition is commutative. Look for an easier order to add the list.

How it Works:

$$42 + 36 + 8$$

Look over the problem for numbers that are easy to add up.

$$42 + 8 + 36$$

$$86$$

Multiply Repeating Addends

The definition of multiplication is repeated addition.

How it Works:

$$21 + 21 + 21 + 21 + 21 + 21$$

$$6 \times 21 = 6 \times 20 + 6 \times 1$$

$$126$$

Add Repeating Digits

When there is a pattern to the digits in the numbers being added, sum up the digits once and reuse that sum.

How it Works:

$$234 + 342 + 423$$

Look over the problem to find a pattern. Here the hundreds, tens and units digits are repeating the same numbers.

$2 + 3 + 4 = 9$ Add the numbers. Remember that addition is commutative.

Use the result in all places.

$$999$$

PRACTICE: ADDING IN OUR HEADS

Day 1:

Draw a line from the addition strategy that you might use to solve each problem

Keep a Running Total $85 + 57 + 15$

$29 + 16 + 21 + 34$

Use an Ordering that Makes Adding Easier $95 + 18 + 42 + 36 + 76$

Try to solve using an ordering that makes adding easier:

$$85 + 57 + 15$$

$$29 + 16 + 21 + 34$$

Try to add simply keeping a running total:

$$32 + 15 + 17 + 21$$

Try to add these lists with a strategy of your choosing:

$$37 + 38 + 22 + 40$$

$$95 + 18 + 42 + 36 + 76$$

$$67 + 98 + 21$$

PRACTICE: ADDING IN OUR HEADS

Day 2:

Draw a line from the addition strategy that you might use to solve each problem

Add Evenly Spaced Numbers $111 + 111 + 111 + 111 + 111$

Add Lists with a Pattern $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$

Add Repeating Digits $123 + 231 + 123$

Try to solve using the evenly spaced numbers strategy:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$$

Try to add this list that has a pattern:

$$123 + 231 + 123$$

Try to add using multiplication:

$$111 + 111 + 111 + 111 + 111$$

Try to add these lists with a strategy of your choosing:

$$1112 + 1121 + 1211 + 2111$$

$$4 + 9 + 14 + 19 + 24 + 29$$

$$21 + 21 + 21 + 21 + 21$$

Day 3:

Try to solve these using the best method that you know.

$$2 + 4 + 6 + \dots + 48 \quad (\text{all the evens from 2 to 48})$$

Take-Away Mental Math

Can you add in your head using the Break-Apart method?
Draw red lines to show where you can break numbers apart.
Check your answers using stacks if unsure of your result.

$$20\overset{|}{1}9 + 19\overset{|}{1}9 =$$

$$14 + 41 =$$

$$921 - 603 =$$

$$85 + 95 =$$

$$18 + 2018 =$$

$$105 + 115 =$$

$$965 - 433 =$$

$$75 + 12 =$$

$$6734 + 125 =$$

$$21 + 22 + 23 =$$

$$435 - 314 =$$

$$343 - 237 =$$

$$2372 + 123 =$$

$$301 + 96 =$$

$$54 + 36 =$$

$$821 + 182 =$$

$$27 + 13 =$$

$$1470 + 1047 =$$

$$59 + 34 =$$

$$73 + 101 =$$

$$23 + 17 =$$

$$623 + 812 + 14 =$$

$$17 + 2017 =$$

$$24 + 52 + 12 =$$

Take-Away Mental Math

Can you add these lists of numbers in your head? Write down which method you used.

Adding In Our Heads Strategies

Keep a Running Total –
Breaking Apart Tens from
Units

Add in Pairs For Evenly
Space Numbers

Multiply Repeating
Numbers

Rely on Friendly Numbers

Use an Easier Order

Add Repeating Digits

$$25 + 25 + 25 + 25 + 25 + 25$$

$$13 + 10 + 10 + 2 + 21$$

$$106 + 98$$

$$27 + 34 + 47 + 24$$

$$1 + 3 + 5 + 7 + 9 + 11$$

$$1232 + 2321 + 3212 + 2123$$

$$52 + 13 + 79$$

$$11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11$$

$$5 + 15 + 25 + 35 + 45$$

$$67 + 97 + 21$$

$$88 + 28 + 12$$

The answers are 150, 56, 204, 132, 36, 888, 144, 99, 125, 185, 128

PRACTICE: RELY ON FRIENDLY NUMBERS

Day 1:

$99 + 99 + 99 + 99$

$24 + 25$

$77 - 45$

$511 - 292$

$435 - 314$

$199 + 13$

$998 + 49$

$249 + 248$

$219 - 198$

$65 - 433$

Day 3:

$452 - 212$

$54 + 43 + 24 + 12$

$310 + 293 + 219 + 113$

$8293 + 1904$

$17 + 2017$

Day 2:

$126 + 124$

$56 + 129$

$301 + 21 + 198$

$211 - 13 - 58$

$106 + 98$

$93 + 114 + 93$

$932 - 486$

$67 + 97 + 21$

$295 + 397 + 108$

$34 + 46 - 23$

$14 + 41$

$21 + 22 + 23$

$13 + 6 - 4 - 5$

$343 - 237$

$32 + 74 - 45$

SUBTRACTING IN OUR HEADS

1. Use Negative Numbers

$$97-68=30$$

$$30-1=29$$

Math talk (kids words) ninety seven minus...

1. Change the starting number so that it is easier to take the other number away.

$$97-68=30$$

$$30-1=29$$

I started one too high, so I have to take one away...

$$90-48=89-48=41+1=42$$

I started one too low, so I have to add one back...

1. Change the take-away number so it is friendly with your starting number.

$$97-68=97-67=30-1=29$$

$$30-1=29$$

I took away too little, so I have to take away the rest.

$$91-68=91-71=20+3=23$$

I took away too much so I have to put some back.