Numbers

A Case for a Deep Understanding of Numbers.

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Document Purpose Numbers

The purpose of this document is to discuss the current philosophy behind learning numbers, why it is an important part of NJLovesMath and how you can help your learner develop a deep understanding of and love for numbers.

The intended audience for this document are the caregivers and coaches of elementary learners of mathematics. This document considers:

- **What is the problem with our understanding of numbers?**
- **What is a number?**
- **How we represent numbers?**
- **Why numbers matter?**
- **How can we help our learners?**

What is the problem with our understanding of numbers?

It is called Arithmetic when we should call it **Numbers**.

Symbolic **mathematics is required of us when we should be exploring physical and visual models**.

We hop across number types; we should be **building up**.

We develop a deep understanding of numbers when we put it on equal footing with arithmetic and build up foundational models within a woven fabric of mathematics.

What is a number?

A number is a point on the number line.

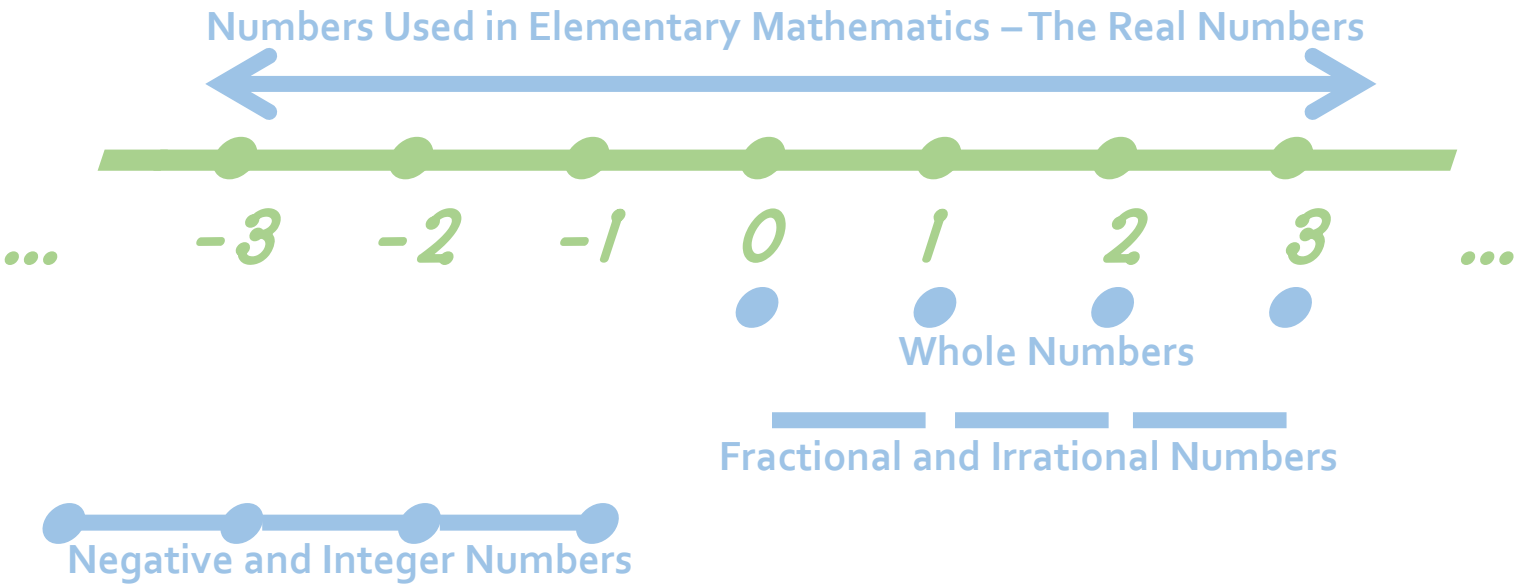
We use this definition because it helps learners build a deeper understanding of numbers throughout elementary school mathematics.

Why this definition is powerful?

A number is a point on the number line.

The number line provides learners with a framework that is flexible enough for starting with counting numbers and building up to whole numbers, fractions and negatives.

The number line corresponds to all of the numbers we need for elementary mathematics. Every whole number, fraction, irrational and their negatives corresponds to a point on the number line. Together these points fill up all of the space on the number line.



The number line is the complete set of all numbers used in elementary mathematics (i.e. wholes, fractionals, irrationals, negatives and integers.)

What is a number? A value …

We use the value of a number to define the **quantity** or **position** of something…

…to help us measure, order or compute on that something.

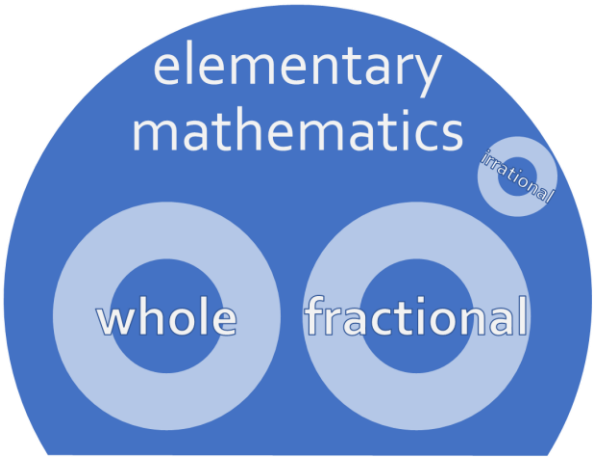
A value is intrinsic to the number. Whether we represent **50** as $\frac{100}{2}$ or 50.0 or $5x10^1$ or L… whether we use numerals, tick marks or display a flock of 50 sheep… in one big group or in 5 groups of 10… the value remains 50.

What is a number? … a type …

"What numbers we need for elementary mathematics is fortunately nothing very sophisticated – just the wholes, the fractions and the negatives, with a few irrational numbers thrown in now and then." – Hung-Hsi Wu

There are three types of numbers used in elementary mathematics.

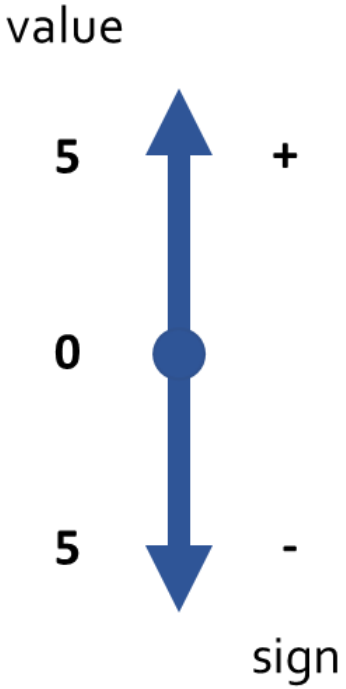
whole fractional *a sprinkle of irrationals*



What is a number? ... and a sign.

Numbers can have an implicit or explicit sign.

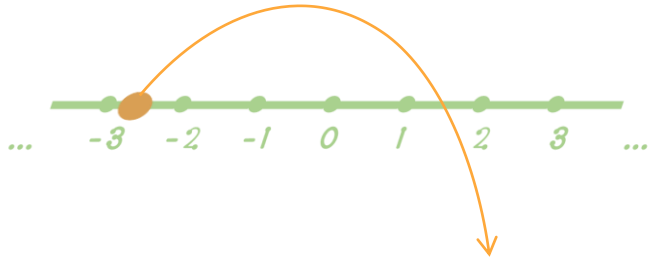
positive negative none both



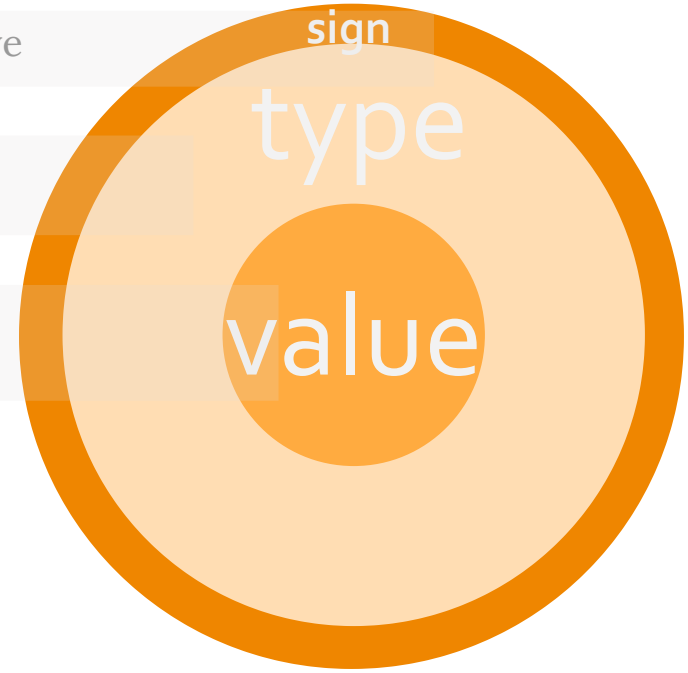
Zero can be considered as having no sign or as being both positive and negative.

So what is a number again?

A point on the number line...



...that has value, type and sign.



positive negative
none
both

whole
fractional
irrational

quantity
position

We do not consider nominals (i.e. labels) to fit the definition of numbers.

More numbers vocabulary...

digit
from the Latin digitus meaning
'finger or toe'.
a symbol that can be used alone or in combinations to represent a number.

numeral
from the french numéral meaning
'word expressing a number'.
a symbol or a group of symbols or a name that stands for a number; how we write it.

ordinal
from the Latin ordo meaning
'row or series. order'.

cardinal
from the Latin cardinalis meaning
'serving as a hinge'.

nominal
from the Latin nominalis meaning
'name'.

natural / counting
those numbers used for counting.

whole / nonnegative
from the Old English hal meaning
'entire'.
those numbers without fractional parts; and no negatives.

prime
from the Latin primus meaning
'principal or noble'.
those numbers measured by a unit alone.

composite
from the Latin compositus meaning
'put together'.

fraction
from the French fraccion meaning
'a breaking'.

real
from the Latin reals meaning
'having an actual existence'.
the type of number we normally use.

rational
from the Latin rationalis meaning
'springing from reason'.
all numbers that are ratios of integers.

irrational
from the Latin irrationalis meaning
'without reason'.
all numbers that cannot be made by dividing two integers.

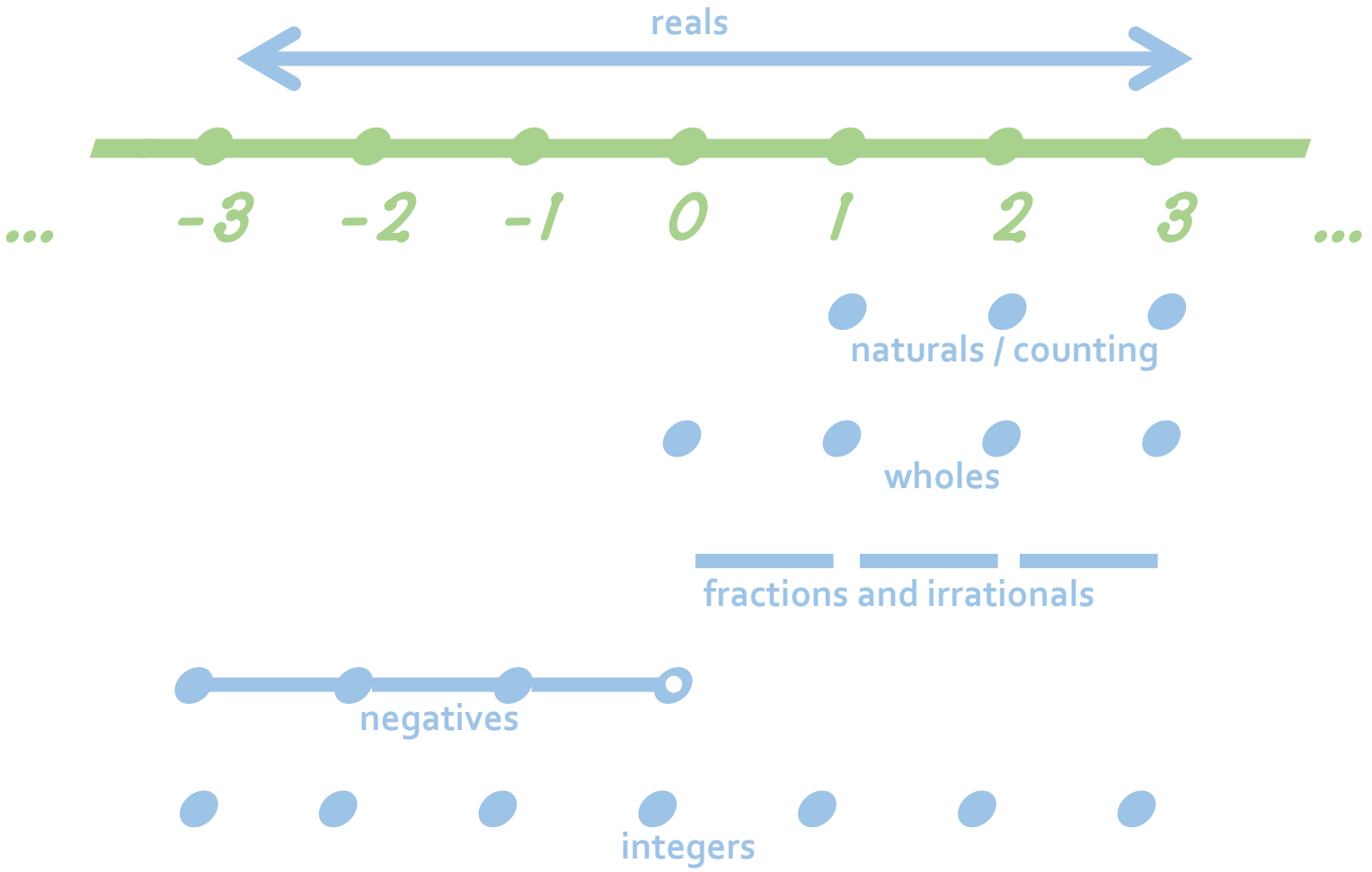
integer
from the Latin integer meaning
'intact'.
all whole and negative numbers; no fractions allowed.

negative
from the Latin negativus meaning
'that which denies'.
less than 0.

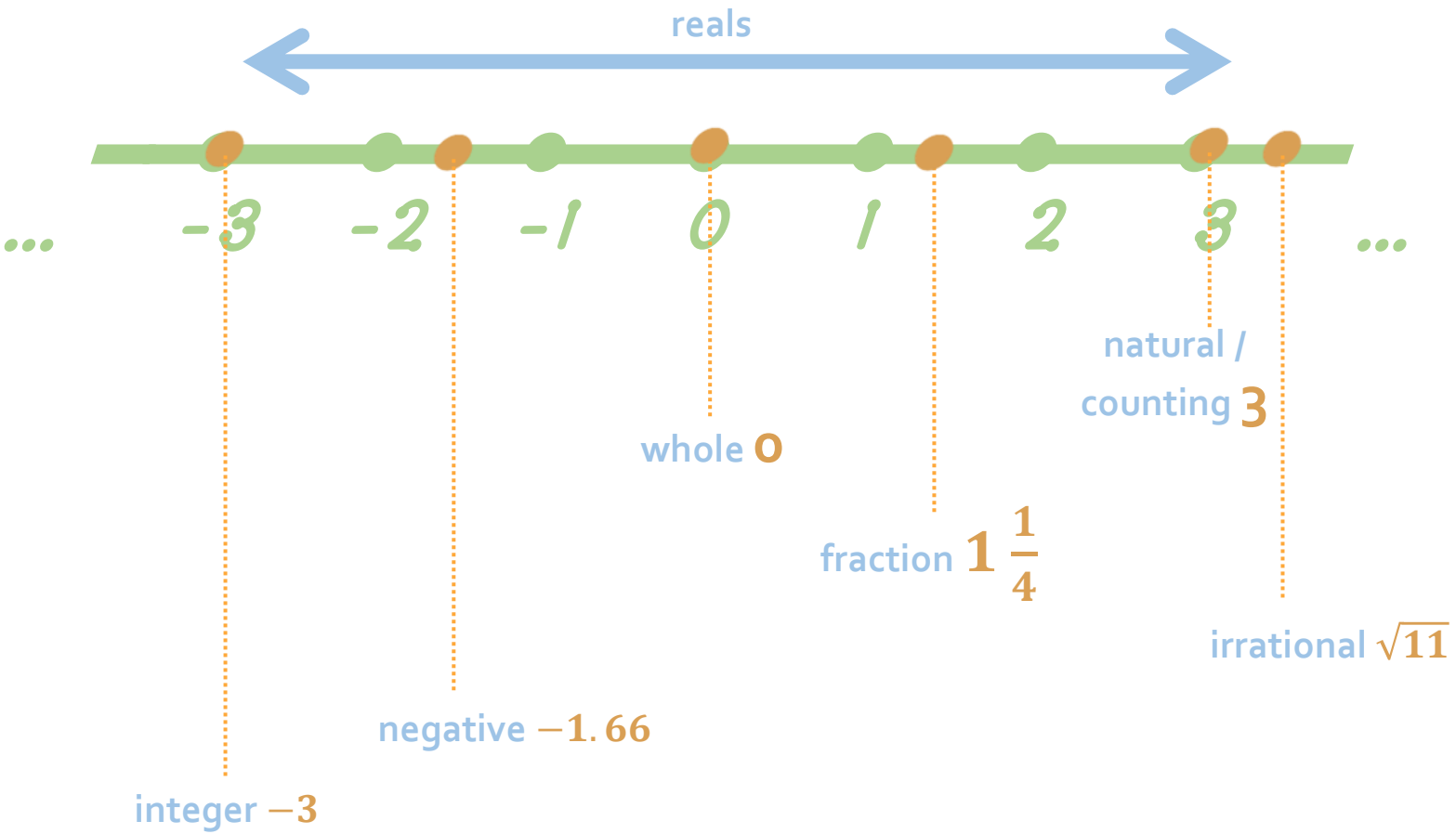
decimal point
from the Latin decimalis meaning
'of tenths'.
a symbol that separates the integer from the fractional part.

decimal system
from the Latin negativus meaning
'of tenths'.
positional number system with 10 as the base.

...to complete the entire number line...



...with examples.



A number is a number is a number...

...no matter how we **represent** the number,

the number retains its place on the number line.

notations
(integer, decimal, expanded, standard, exponential, scientific, fraction-proper, improper, irreducible, reducible, percent, ...)

number systems
(base 2-computers, base 10-metric, base 16-colors, addresses, base 12-factorable, roman-beauty...)

modelling
(physical, iconic, symbolic..)

quantity, measure
(numbers become quantities with the inclusion of units - metric, US standard, temperature, weight, time...)

estimated, rounded, approximated,

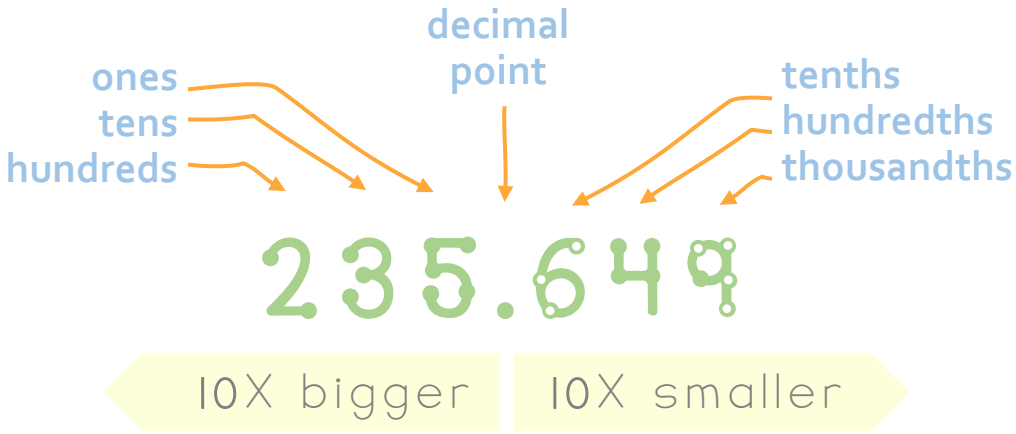
represent

number

Irrespective of how we choose to represent a number,
the number retains its position on the number line.

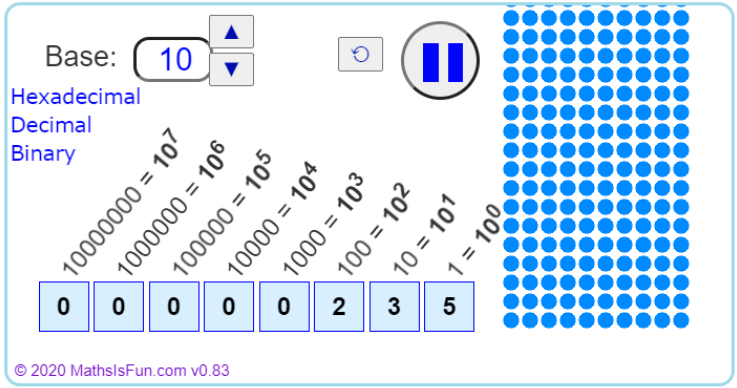
How We Represent a Number in the Decimal System?

A positional system ... its all about the place value.



A base 10 system ... it has 10 digits.

0 1 2 3 4 5 6 7 8 9



Why Numbers Matter?

We avoid the hard work.

We remove the hard work of mathematics when we learn number sense.

Composing and decomposing numbers (e.g. factors and multiples, $25 = 2 \times 10^1 + 5 \times 10^0 = 5 \times 5 = \frac{100}{4}$.)

Estimating, rounding and approximating.

Equivalent representations (e.g. decimals for fractions or vice versa.)

Scientific Notation (e.g. to work with very large and very small numbers.)

Recognizing characteristics of sets of numbers (e.g. properties of evens, divisibility rules, primes and composites, sequences and series.)

Other number systems (e.g. binary, hexadecimal, large.)

Other measurement systems (e.g. standard, which uses base 12 and 20, to metric, which uses the easier base 10.)

As we become familiar with all of the properties of numbers available to us, we use our number sense to remove the hard work of computing.

Why Numbers Matter? Mathematicians and Leading Educators say so.

Learning is a gradual process with each step firmly rooted in prior experiences; whole numbers lead us to fractions and visual representations (number lines) lead us from the physical world to the symbolic. Learners need concrete images to anchor the many concepts they are learning (Charlotte Mason, Hunghzi Wu).

Learners can think fractions are scary beasts; but fractions and whole numbers are on equal footing, a natural extension of whole numbers, then our chances of success in teaching fractions should increase immeasurably. For then, children will gain the confidence that they can use what they know about whole numbers as a guide in this new venture (Charlotte Mason, Hunghzi Wu).

Successful mathematicians in history (Charles Gauss) and today (Terry Tao, James Maynard) assert that their findings and proofs are all built from their strong sense of numbers and understanding of number theory.

Using a number line approach to learning numbers provides learners the confidence that they can use what they know about whole numbers as a guide to learning more complex numbers.

How We Can Help Our Learners?

Math Club is 1 hour a week; of that, 10 minutes are spent on Number Sense. Number Sense concepts could use additional reinforcement. The club could use your help:

- ➔Number Sense problems will be made available weekly
  Help learners discuss the numbers in their answers, their type, value, sign and where they fall on the number line; how are the numbers being represented and why? Did the representation make it easier or harder to work with the particular number.

- ➔Spend 10 minutes solving Number Sense problems
  a couple of days a week. Draw a line or remember the page where you and your child left off. Reflect on the more challenging questions. No pencils, no paper and no stress.

- ➔Spend 10 minutes solving the shortcut of the week
  a couple of days a week, alternating with Number Sense. A packet is provided in your child's

Please no worries if you cannot!

The Math Club will always be working on Number Sense.
These strategies will become available on our Math Club blog.

Start the drumbeat if or when it makes sense for you and your learner.

References and Further Reading

Charlotte Mason (1928). Number: A Figure and a Step Onward.

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